(55) Find $\underset{x\to\frac{\pi}{2}}{Lt} (\sin x)^{\tan x}$.

Ans. → $\underset{x\to\frac{\pi}{2}}{Lt} (\sin x)^{\tan x} \quad [1^{\infty}]$

Let $y = \underset{x\to\frac{\pi}{2}}{Lt} (\sin x)^{\tan x}$

Taking log both sides, we have,

$$\log_e^y = \underset{x\to\frac{\pi}{2}}{Lt} \tan \cdot \log \sin x$$

$$= \underset{x\to\frac{\pi}{2}}{Lt} \frac{\log \sin x}{\cot x} \left[\frac{0}{0}\right]$$

Hence, from L' Haspital Rule, we have

$$\log_e y = \underset{x \to \frac{\pi}{2}}{Lt} \frac{\frac{1}{\sin x} \times \cos x}{-\operatorname{cosec}^2 x}$$

$$= \underset{x \to \frac{\pi}{2}}{Lt} \frac{\frac{\cos x}{\cancel{\sin x}}}{-\frac{1}{\sin^{\cancel{2}} x}}$$

$$= \underset{x \to \frac{\pi}{2}}{Lt} - \sin x \cdot \cos x$$

$$= 1 \times 0 = 0$$

$\because \log_e y = 0$

$\therefore y = e^0 = 1$ Ans.

(56) Evaluate $\underset{x \to \frac{\pi}{2}}{Lt} (\sin x)^{2\tan x}$.

Ans. $\to \underset{x \to \frac{\pi}{2}}{Lt} (\sin x)^{2\tan x} \quad [1^{\infty}]$

Let $y = \underset{x \to \frac{\pi}{2}}{Lt} (\sin x)^{2\tan x}$

Taking log both sides, we have,

$$\log_e y = \underset{x \to \frac{\pi}{2}}{Lt} 2\tan x \cdot \log \sin x$$

$$= \underset{x \to \pi/2}{Lt} \frac{\log \sin x}{2\cot x} \left[\frac{0}{0}\right]$$

Hence, from L' Hospital's Rule, we have

$$\log_e y = \underset{x \to \frac{\pi}{2}}{Lt} \frac{\frac{1}{\sin x} \times \cos x}{-2\operatorname{cosec}^2 x}$$

$$= \underset{x \to \frac{\pi}{2}}{Lt} \frac{\frac{\cos x}{\sin x}}{-2 \cdot \frac{1}{\sin^2 x}}$$

$$= \underset{x \to \frac{\pi}{2}}{Lt} -2\sin x \cdot \cos x$$

$$= -2 \times 0 = 0$$

$\therefore \log_e y = 0$

$\therefore y = e^0 = 1$ Ans.

(58) Evaluate $\underset{x \to a}{Lt} \left(2 - \frac{x}{a}\right)^{\tan \frac{\pi x}{2a}}$.

Ans. → $\underset{x \to a}{Lt} \left(2 - \frac{x}{a}\right)^{\tan \frac{\pi x}{2a}}$ $[1^\infty]$

Let $y = \underset{x \to a}{Lt} \left(2 - \frac{x}{a}\right)^{\tan \frac{\pi x}{2a}}$

Taking log both sides, we have,

$$\log_e y = \underset{x \to a}{Lt} \tan \frac{\pi x}{2a} \cdot \log\left(2 - \frac{x}{a}\right)$$

$$\log_e y = \underset{x \to a}{Lt} \frac{\log\left(2 - \frac{x}{a}\right)}{\cot \frac{\pi x}{2a}} \quad \left[\frac{0}{0}\right]$$

Hence from, L' Hospital's Rule, we have,

$$\log_e y = \underset{x \to a}{Lt} \frac{\frac{1}{2 - \frac{x}{a}} \times -\frac{1}{a}}{-\frac{\pi}{2a} \cdot \operatorname{cosec}^2 \frac{\pi x}{2a}}$$

$$= \frac{1}{2 - \frac{a}{a}} \times \cancel{-}\frac{1}{\cancel{a}} \times \cancel{-}\frac{2\cancel{a}}{\pi} \cdot \frac{1}{\operatorname{cosec}^2 \frac{\pi a}{2a}}$$

$$= \frac{1}{\frac{2a-a}{a}} \times \frac{2}{\pi} \cdot \frac{1}{\operatorname{cosec}^2 \frac{\pi}{2}}$$

$$= \frac{1}{\frac{\cancel{a}}{\cancel{a}}} \times \frac{2}{\pi} \cdot (1)^2 = 2\pi$$

$\therefore \log_e y = 2\pi$

$\therefore y = e^{2\pi}$ Ans.

(59) Evaluate $\underset{x \to 0}{Lt} \left(\frac{\sin x}{x}\right)^{1/x}$.

Ans. → Let $y = \underset{x \to 0}{Lt} \left(\frac{\sin x}{x}\right)^{1/x} \quad [1^\infty]$

Taking log both sides, we have,

$$\log_e y = \underset{x \to 0}{Lt} \frac{1}{x} \log\left(\frac{\sin x}{x}\right)$$

$$= \underset{x \to 0}{Lt} \frac{\log \frac{\sin x}{x}}{x} \quad \left[\frac{0}{0}\right]$$

$$\log_e^y = \underset{x\to 0}{Lt} \frac{\log \sin x - \log x}{x} \quad \left[\frac{0}{0}\right]$$

$$= \underset{x\to 0}{Lt} \frac{\frac{1}{\sin x}\cdot \cos x - \frac{1}{x}}{1}$$

$$= \underset{x\to 0}{Lt} \frac{\frac{\cos x}{\sin x} - \frac{1}{x}}{1} = \underset{x\to 0}{Lt} \frac{\frac{x\cos x - \sin x}{x \sin x}}{1}$$

$$= \underset{x\to 0}{Lt} \frac{x\cos x - \sin x}{x \sin x} \quad \left[\frac{0}{0}\right]$$

$$= \underset{x\to 0}{Lt} \frac{1\times \cancel{\cos x} - x\sin x - \cancel{\cos x}}{1\times \sin x + x\cos x} \quad \left[\frac{0}{0}\right]$$

$$= \underset{x\to 0}{Lt} \frac{-1\times \sin x - x\cos x}{\cos x + 1\times \cos x - x\sin x}$$

$$= \underset{x\to 0}{Lt} \frac{-\sin x - x\cos x}{\cos x + \cos x - x\sin x}$$

$$= \frac{0-0}{1+1-0} = \frac{0}{2} = 0$$

$\therefore \log_e^y = 0$

$\therefore y = e^0 = 1$ Ans.